Phys514 Fall 2013: HW5 Solution

TA: David Chen*

Note: N will denote particle number and n particle density

1 Foot 9.7 – Optical molasses damping (25 pts)

Part a. From Foot 9.15, $F_{molasses} = F_{sc}(\delta - kv) - F_{sc}(\delta + kv)$, where $F_{sc}(\delta) = \hbar k \frac{\Gamma}{2} \frac{s}{1+s+4\delta^2/\Gamma^2}$ and $s \equiv I/I_{sat}$. We can approximate the peak force of $F_{molasses}$ by $F_{sc}(0) = \hbar k \frac{\Gamma}{2}\frac{s}{1+s}$. The slope of $F_{molasses}$ at $v = 0$ is $\alpha \equiv |\frac{d}{dv}F_{molasses}| = 2k\frac{\partial F_{sc}}{\partial \delta} = 8\hbar k^2 \frac{|\delta|}{\Gamma}\frac{s}{(1+s+4\delta^2/\Gamma^2)^2}$. For the particular case of $\delta = -\frac{\Gamma}{2}$, $\boxed{\alpha = 4\hbar k^2 \frac{s}{(s+2)^2}}$. Note that $\frac{\alpha}{F_{sc}(0)} = \frac{8k}{\Gamma}\frac{s+1}{(s+2)^2} \approx \frac{2k}{\Gamma}$ for $s \ll 1$, which is consistent with the problem statement.

Part b. If $I = I_{sat}$ and $\lambda \approx 589$ nm, then $\tau = \frac{M}{2\alpha} = \boxed{3.6\ \mu s}$

2 Foot 9.11 – Equilibrium MOT atom number (25 pts)

Part a. The 3D Maxwell-Boltzmann distribution is given by $f(v) = \frac{4}{\sqrt{\pi}v_p^3}v^2 e^{-v^2/v_p^2}$, where $v_p \equiv \sqrt{\frac{2k_BT}{m}}$. Particles enter the MOT at a rate $\dot{N}_{in} = \frac{1}{4}nA\int_0^{v_c} vf(v)dv \approx \frac{1}{4}nA\frac{4}{\sqrt{\pi}v_p^3}\int_0^{v_c} v^3 dv = \boxed{\frac{nAv_c}{4\sqrt{\pi}}(v_c/v_p)^3}$, where n is the background density.

Part b. Atoms are removed from the MOT at a rate $\dot{N}_{out} = -n\bar{v}\sigma N$, where $\bar{v} = 2v_p/\sqrt{\pi}$ and σ is the atom-atom collision cross-section. At equilibrium, $\dot{N}_{in} = \dot{N}_{out}$, then the atom number is $\boxed{N = \frac{A}{8\sigma}(v_c/v_p)^4}$. N is independent of the background density n and, therefore, the pressure is $P = nk_BT$.

Part c. If $v_c = 25$ m/s, $D = 2$ cm and $\sigma \sim \pi\lambda_T^2 = \frac{h^2}{2\pi k_B T_{mot} m}$, where λ_T is the thermal de Broglie wavelength and T_{mot} is the temperature of the MOT, on the order of the Doppler cooling limit $\frac{\hbar\Gamma}{2k_B} \sim 150\ \mu$K, then $\boxed{N \sim 10^7 - 10^8}$

3 Foot 9.12 – Absorption of atoms trapped in a MOT (25 pts)

Part a. $I = I_0 e^{-n\sigma(\omega)\Delta z} \approx I_0(1 - n\sigma(\omega)\Delta z)$, then $\frac{\Delta I}{I_0} = n\sigma(\omega)\Delta z = \frac{N\sigma(\omega)2r}{4\pi r^3/3} \approx \boxed{\frac{N\sigma(\omega)}{2r^2}}$

Part b. From Foot 7.76 and 7.80, $\sigma(\omega) = \sigma_0 \frac{\Gamma^2/4}{\delta^2+\Gamma^2/4}$, where $\sigma_0 = \frac{3\lambda_0^2}{2\pi} = 2.9 \cdot 10^{-13}$ m^2. At $\delta = -2\Gamma$, $\sigma = \sigma_0/17 = 1.7 \cdot 10^{-14}$ m^2. Therefore $r = \sqrt{\frac{N\sigma}{2}} = \boxed{3\text{ mm}}$ and $n = N/(\frac{4\pi r^3}{3}) = \boxed{9.5 \cdot 10^{15}\ \text{m}^{-3}}$

*dchen30@illinois.edu

4 Foot 9.16 – Simple optical lattice (25 pts)

The lattice potential is given by $U = U_0 \cos^2 kx = \frac{U_0}{2}(1 + \cos 2kx)$. The oscillation frequency is $\omega = \sqrt{\frac{U''(x_{min})}{m}} = \sqrt{\frac{2k^2}{m} U_0}$. If we use the parameters $k = \frac{2\pi}{1060\text{nm}}$, $m = \frac{23 \text{ kg}}{6.03 \cdot 10^{26}}$, $U_0 = 100\ E_r$, $E_r = \frac{\hbar^2}{2m}(\frac{2\pi}{\lambda_0})^2$ and $\lambda_0 = 589$ nm, we obtain $\omega = \boxed{(2\pi)\ 277 \text{ kHz}}$ and the energy spacing $\Delta E = \hbar\omega = \boxed{277 \text{ kHz} \cdot h}$